1. Introduction
Underground energy storage is the main option for large scale energy storage, apart from pumped-hydro.

Modeling challenges:
- large domains and limited data,
- locally complex processes,
- dynamic boundary conditions.

Here, we present a coupled model that applies:
- a full-dimensional model in regions of higher complexity and where the vertical equilibrium assumption does not hold;
- a vertical equilibrium model in the rest of the domain.

2. Existing Models
2.1 Full-dimensional model:
- Mass balance equation:
  \[
  \frac{d}{dt}(\rho_{\text{ni}} + \rho_{\text{nw}}) + \nabla \cdot (\rho_{\text{ni}} \mathbf{u}_{\text{ni}} + \rho_{\text{nw}} \mathbf{u}_{\text{nw}}) = \rho_{\text{ni}} \mathbf{q}_{\text{ni}} + \rho_{\text{nw}} \mathbf{q}_{\text{nw}},
  \]
  Darcy’s law:
  \[
  \mathbf{u}_{\text{ni}} = -K_{\text{ni}} \nabla (p_{\text{ni}} - \phi \mathbf{g}),
  \]
  with wetting/non-wetting phase \(\phi\), saturation \(s\), pressure \(p\), density \(\rho\), permeability tensor \(K\), relative permeability \(K_{\text{r}}\), porosity \(\phi\), and gravitational acceleration \(g\).

2.2 Vertical equilibrium model:
- Mass balance equation:
  \[
  \frac{d}{dt}(\rho_{\text{ni}} S_{\text{ni}} + \rho_{\text{nw}} S_{\text{nw}}) + \nabla \cdot (\rho_{\text{ni}} \mathbf{u}_{\text{ni}} S_{\text{ni}} + \rho_{\text{nw}} \mathbf{u}_{\text{nw}} S_{\text{nw}}) = \rho_{\text{ni}} \mathbf{q}_{\text{ni}} S_{\text{ni}} + \rho_{\text{nw}} \mathbf{q}_{\text{nw}} S_{\text{nw}},
  \]
  Darcy’s law:
  \[
  \mathbf{u}_{\text{ni}} = -K_{\text{ni}} (S_{\text{ni}} p_{\text{ni}} - \phi \mathbf{g}),
  \]
  with vertically integrated variables and reference pressure.

3. Model Coupling
- Discretized mass balance equation (Finite Volume Method):
  \[
  \sum_i q_{\text{ni},i} = \sum_{j=1}^{N_{\text{R}}} q_{\text{nj},j} = q_{\text{ext},j},
  \]
  with source/sink \(q_{\text{ext}}\).
- Total velocity from VE-cell \(i\) to 2D cell \(j\):
  \[
  \mathbf{v}_{\text{ij}} = -K_{\text{ij}} \left( \frac{\Delta p_{\text{ij}}}{\Delta z_{\text{ij}}} + \mathbf{L}_{\text{ij}} \mathbf{S}_{\text{ij}} \right).
  \]
- Reconstructed pressures in VE ghost cells:
  \[
  p_{\text{ri}}^\text{V} = \bar{p}_{\text{n},i} - (\gamma \Delta z),
  p_{\text{n},i} = \bar{p}_{\text{n},i},
  \]
  Calculation of secondary variables in VE ghost cells:
  Total mobility \(K_{\text{V}} = \lambda + \lambda_{\text{r}}\) and fractional flow function \(\gamma = \lambda^{-1}/K_{\text{V}}\)
  based on averaged saturation in ghost cell saturation \(\bar{s}_{\text{n},i}\).
- IMPES-algorithm: saturation \(S_{\text{n}}\) in VE-cell, reconstructed saturation \(\bar{s}_{\text{n}}\) and saturation in ghost cell \(s_{\text{n}}\) based on old time step.

4. Criterion for VE Applicability
- Local criterion to quantify conformity of the full-dimensional solution with the vertical equilibrium assumption.
- Calculated as the area between profiles (saturation or relative permeability) over 2 for each column.
- Full-dimensional profile determined based on simulation results, VE profile determined based on average saturation in column.

5. Results
Brooks-Corey cap. pressure:
\[
\lambda = 2.0, \quad \rho_0 = 1 \times 10^5 \text{ Pa}
\]
Phase properties (CH<sub>4</sub>, water):
\[
\rho_1 = \begin{cases} 59 \text{ kg/m}^3 & \text{CH}_4 \\ 1000 \text{ kg/m}^3 & \text{water} \end{cases},
\]
\[
\mu_1 = \begin{cases} 1.2 \times 10^{-5} \text{ Pa s} & \text{CH}_4 \\ 5 \times 10^{-3} \text{ Pa s} & \text{water} \end{cases}
\]
Injection rate: \(Q_{\text{in}} = 552 \text{ t/m/a}\)

6. Summary and Outlook
Summary:
- A coupled model of VE and full-dimension is developed.
- A criterion for applicability of the VE model is developed and tested.
- The coupled model reduces computational effort significantly while maintaining accuracy.

Outlook:
- Implement adaptive boundary between model domains. Test adaptation criteria.
- Analysis of advantages and disadvantages of adaptive concept.
- Include hysteresis in the model.
- Test concept for field scale case of underground energy storage.

7. References
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