Modeling challenges:
- large domains and limited data,
- locally complex processes,
- dynamic boundary conditions.

Here, we present a coupled model that adaptively applies:
- a full-dimensional model in regions of higher complexity and where the vertical equilibrium assumption does not hold,
- a vertical equilibrium model in the rest of the domain.

State of current work

1. Models

1.1 Full-dimensional model:
- Mass balance equation:
  \[ \nabla \cdot (\rho_s \phi_s \mathbf{u}) = \nabla \cdot (\rho_l \phi_l \mathbf{w}) + q \],
- Darcy’s law:
  \[ \mathbf{u} = -K \nabla p \]
with wetting/non-wetting phase \( \rho_s \), density \( \phi_s \), pressure \( \phi_s \), porosity \( \phi \), relative permeability \( K \), viscosity \( w \), sink/source \( q \).

1.2 Vertical equilibrium model:
- Mass balance equation:
  \[ \frac{1}{r} \frac{d}{dr} \left( r \frac{d}{dr} (r \phi_s \mathbf{u}) \right) = \frac{1}{r} \frac{d}{dr} (r \phi_l \mathbf{w}) + q \]
- Darcy’s law:
  \[ \mathbf{u} = -K \frac{d}{dr} \left( r \phi \right) \frac{d}{dr} \frac{dr}{r} \]
with vertically integrated variables and reference pressure.

2. Model coupling

- Discretized mass balance equation (Finite Volume Method):
  \[ \sum_{j=1}^{n} \mathbf{u}_{ij} \cdot \mathbf{n}_{ij} \Delta A_{ij} = \sum_{j=1}^{n} \mathbf{w}_{ij} \cdot \mathbf{n}_{ij} \Delta A_{ij} + q \]
- Total velocity from VE-cell i to 2D cell j:
  \[ \mathbf{U}_{ij} = -K \frac{d}{dr} \frac{dr}{r} \]
- Reconstructed pressures in VE ghost cells:
  \[ p_{ij} = p_{ij} \]
- Calculation of secondary variables in VE ghost cells:
  Total mobility \( \lambda_p \) and fractional flow function \( f_i - f_0 = f_i / f_0 \) based on averaged saturation in ghost cell saturation \( f_i \).

3. Model adaptation

The criterion is normalized by the height \( H_{VE} \) of the VE gas plume:
\[ c_{flat} = \frac{H_{VE}}{H_{VE} < c_f} \]
A buffer zone is introduced between the subdomains.

4. Results

Brooks-Corey cap. pressure:
\[ \lambda = 2.0, \mu_w = 1 \text{ bar} \]
Phase properties (CH\(_4\), water):
\[ \rho_w = 59.2 \text{ kg/m}^3 \]
\[ \mu_w = 911 \text{ kg/m/s} \]
\[ \mu_w = 1.2 \times 10^{-5} \text{ Pa s} \]
Injection rate: \( Q_{inj} = 552 \text{ t/m}^3/a \)

5. Outlook

- Analysis of advantages and disadvantages of adaptive concept.
- Include hysteresis in the model.
- Test concept for field scale case of underground energy storage.

References
Geological Storage of CO\(_2\).