Motivation

Setup of an infiltration experiment. At the top of a sand column having the height \( L \) water is injected, resulting in a water saturation \( S_w^L \) at \( z = 0 \):

\[
\begin{align*}
\text{homogeneous porous medium } & \mathbf{K}, \phi \\
\text{infiltration rate} & \mathbf{v}_w
\end{align*}
\]

- Measurements of water saturations at different depths of the sand column reveal that the water saturation curves exhibit overshoots containing a plateau [2, 3].
- The overshoot region consists of an imbibition front and a drainage front, inbetween these fronts a plateau is formed.

Central issues:

1. How can the standard two-phase flow equations for porous media be extended such that non-monotonous saturation profiles can be simulated?

Wide-spread approach: Extend the standard capillary pressure relationship by a dynamic \( \tau \)-term [1]:

\[
\frac{\partial S_w}{\partial \tau} = \rho_i - \rho_w - \rho_c
\]

2. How is the behavior of the saturation overshoot depending on \( S_w^L \)?

Mathematical methods:

1. Numerical methods: Discretize the standard two-phase flow equations in 1D using finite volumes and implicit euler:

\[
\begin{align*}
\frac{\partial S_w}{\partial \tau} & \mathbf{v} = \frac{\partial K_w}{\partial \tau} (\mathbf{v} - \mathbf{v}_w) + \mathbf{v}_w - \mathbf{v}_c \\
\frac{\partial S_w}{\partial \tau} & \mathbf{v} = \frac{\partial K_m}{\partial \tau} (\mathbf{v} - \mathbf{v}_m) + \mathbf{v}_m - \mathbf{v}_c \\
\frac{\partial S_w}{\partial \tau} & \mathbf{v} = \frac{\partial K_p}{\partial \tau} (\mathbf{v} - \mathbf{v}_p) + \mathbf{v}_p - \mathbf{v}_c
\end{align*}
\]

Create an overshoot not by a \( \tau \)-term, but by a time dependent boundary condition at \( z = 0 \):

\[
S_w(z = 0, t) = \begin{cases} 
S_w^L, & t \leq 500 s \\
S_w^H, & t > 500 s
\end{cases}
\]

In order to account for imbibition and drainage processes, we introduce for each process a constitutive relation for the relative permeabilities as well as the capillary pressure: \( K_w, K_m, K_p, \rho_c, i \in \{im, dr\} \). A transition between the two processes is described by means of smooth scanning curves (hysteresis model).

2. Analytical methods: Assume that the total velocity \( \mathbf{v}_t = \mathbf{v}_w + \mathbf{v}_c \) is constant in space and time and derive a fractional flow formulation from the two-phase flow equations:

\[
\frac{\partial S_w}{\partial \tau} = \frac{\mathbf{v}_t}{S_w - S_w^0} - \frac{\mathbf{v}_m}{S_m - S_w^0} c_{im}(S_w^0) - \frac{\mathbf{v}_w}{S_w - S_w^0} c_{dr}(S_w^0)
\]

This formulation is the basis for estimating the speed \( c \) of the imbibition and drainage front by means of the Rankine-Hugoniot condition [4, 5]:

\[
S_w = \frac{\mathbf{v}_t}{S_w - S_w^0} c_{im}(S_w^0) - \frac{\mathbf{v}_w}{S_w - S_w^0} c_{dr}(S_w^0)
\]

Results:

Variation of \( S_w^L \) leads to different front propagation behavior, which is analyzed for \( S_w^L \in \{0.44, 0.48754, 0.51\} \):

Case 1: \( S_w^L = 0.44, c_{im} < c_{dr}, t \in \{1000\} \), initial plateau vanishes and a second one is created at a lower level

Case 2: \( S_w^L = 0.48754, c_{im} = c_{dr}, t \in \{800\} \), initial plateau is stable

Case 3: \( S_w^L = 0.51, c_{im} > c_{dr}, t \in \{800\} \), initial plateau is enlarged

Conclusion:

The qualitative behavior of the simulated saturation overshoots is in accordance with the analytical considerations.

References